Lesson 12

Quadrilaterals

Use **Cordova Smart Class Software** on the **smart board** in class to make learning enjoyable. [Digital Learning — NEP Guidelines]

Learning Outcomes

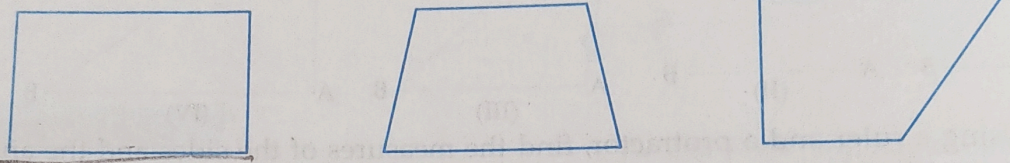
After completing this chapter, you will be able to :

- ❒ define a quadrilateral and name its sides, vertices, angles and diagonals.
- ❒ identify the adjacent and opposite sides of a quadrilateral.
- ❒ identify the interior and exterior of a quadrilateral.
- ❒ differentiate between convex and concave quadrilaterals.
- ❒ define the angle sum property of a quadrilateral and apply it in solving problems.
- ❒ identify various types of quadrilaterals.

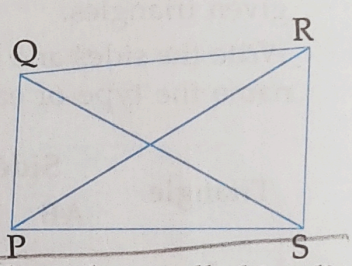
QUADRILATERALS

A simple closed figure bounded by four line segments is called a **quadrilateral**.

The given figures are quadrilaterals as they are closed and have four line segments.



Let us consider a quadrilateral PQRS.



A quadrilateral PQRS has :

(*i*) four **vertices**, namely P, Q, R and S.

(*ii*) four **sides**, namely PQ, QR, RS and SP.

(*iii*) four **angles**, namely ∠QPS, ∠PSR, ∠SRQ and ∠RQP.

(*iv*) two **diagonals**, namely PR and QS.

The line segments joining the opposite vertices of a quadrilateral are called its **diagonals**.

Two diagonals divide the quadrilateral into four triangles.

Adjacent Sides

In quadrilateral PQRS, two sides PQ and QR having a common end point Q are adjacent sides.

Similarly, QR and RS, RS and SP, SP and PQ are adjacent sides of the quadrilateral PQRS.

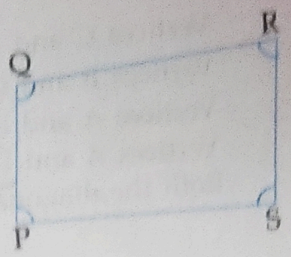
Opposite Sides

The sides PQ and SR are opposite sides. Similarly, PS and QR are opposite sides.

The opposite sides do not have any common end point.

Adjacent Angles

Two angles of a quadrilateral having a common side are called its adjacent angles. $\angle P$ and $\angle S$ are adjacent angles having common side PS. Similarly, $\angle S$ and $\angle R$, $\angle R$ and $\angle Q$, $\angle Q$ and $\angle P$ are adjacent angles of quadrilateral PQRS.



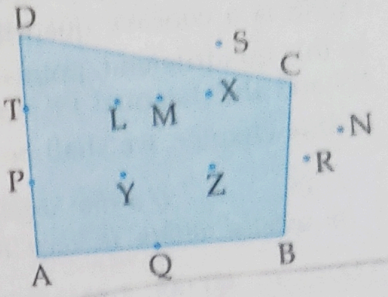
Opposite Angles

Two angles of a quadrilateral that have no common arm are called opposite angles.

In quadrilateral PQRS, $\angle P$ and $\angle R$ are opposite angles. Similarly, $\angle Q$ and $\angle S$ are opposite angles.

Interior and Exterior of a Quadrilateral

Consider a quadrilateral ABCD. It is a plane figure. All points in the plane of the quadrilateral ABCD are divided into three parts :



Interior Region : The region inside the quadrilateral is called the interior of the quadrilateral. Each point of this part is called an interior point of the quadrilateral. In the figure L, M, X, Y, Z are interior points of the quadrilateral.

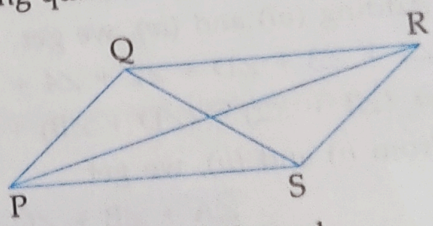
Exterior Region : The part of the plane lying outside the boundary ABCD is called the exterior region of the quadrilateral. Each point of this part is called an exterior point of the quadrilateral. In the figure, R, S, N are exterior points.

On the boundary : The points T, P, Q which lie on the boundary are said to be on the quadrilateral.

Quadrilateral Region

The interior of the quadrilateral ABCD together with its boundary is called the quadrilateral region ABCD.

Example 1 : Name the sides, angles and vertices of the following quadrilateral. What are the names for line segments PR and QS?



Solution : In quadrilateral PQRS,

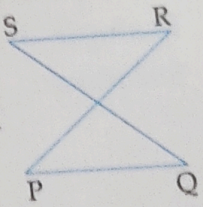
Vertices are P, Q, R and S.

Sides are PQ, QR, RS and SP.

Angles are $\angle QPS$, $\angle PSR$, $\angle SRQ$ and $\angle RQP$.

The line segment PR and QS are diagonals of the above quadrilateral.

Example 2 : Is the adjacent figure a quadrilateral? Give reason.



Solution : The given figure consists of four line segments PQ, QS, PR and SR. Since these line segments intersect at a point other than their end points, therefore, figure is not a quadrilateral.

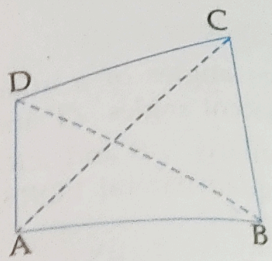
CONVEX AND CONCAVE QUADRILATERALS

Convex Quadrilateral

A quadrilateral is called a **convex quadrilateral**, if the measure of its each angle is less than 180°.

In a convex quadrilateral, the line containing any side of the quadrilateral has the remaining vertices on the same side of it.

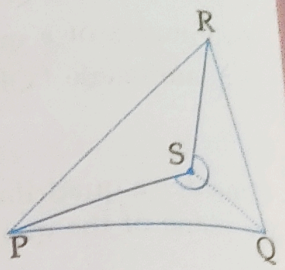
Vertices C and D lie on the same side of the line AB.
Vertices B and C lie on the same side of the line AD.
Vertices A and B lie on the same side of the line CD.
Vertices A and D lie on the same side of the line BC.
Both the diagonals AC and BD lie wholly in the interior.



Concave Quadrilateral

A quadrilateral is called a **concave quadrilateral** if the measure of at least one of the angles is a reflex angle (more than 180°).

PQRS is a concave quadrilateral since, $\angle S > 180°$.



In this quadrilateral, points P and Q lie on the opposite sides of the line RS and the diagonal PR does not lie in its interior.

In this chapter, we shall be dealing only with convex quadrilaterals.

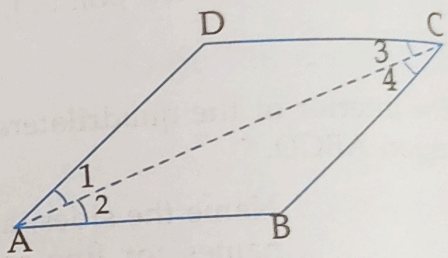
Remember : *If a line segment joining any two points in the interior of a quadrilateral does not lie wholly within it, then it is a concave quadrilateral, otherwise it is a convex quadrilateral.*

ANGLE SUM PROPERTY OF A QUADRILATERAL

Consider the quadrilateral ABCD.

Draw diagonal AC.

Diagonal AC divides the quadrilateral into two triangles *i.e.*, ΔADC and ΔABC.



Now, $\angle A = \angle 1 + \angle 2$ ...(*i*)

$\angle C = \angle 3 + \angle 4$ ...(*ii*)

We know that the sum of the angles of a triangle is 180°.

$\therefore$ In Δ ADC, $\angle 1 + \angle 3 + \angle D = 180°$ ...(*iii*)

In ΔABC, $\angle 2 + \angle 4 + \angle B = 180°$ ...(*iv*)

Adding (*iii*) and (*iv*), we get

$\angle 1 + \angle 3 + \angle D + \angle 2 + \angle 4 + \angle B = 180° + 180°$

$\Rightarrow (\angle 1 + \angle 2) + (\angle D + \angle B) + (\angle 3 + \angle 4) = 360°$

From (*i*) and (*ii*), we get

$$\angle A + \angle B + \angle C + \angle D = 360°$$

Hence, the sum of the angles of a quadrilateral is 360°.

Example 3 : If three angles of a quadrilateral are 100°, 80° and 110°, find the fourth angle.

Solution : Let the fourth angle be x.

Since, the sum of four angles of a quadrilateral is 360°.

$\therefore \quad x + 100° + 80° + 110° = 360°$

$\Rightarrow \quad x + 290° = 360°$

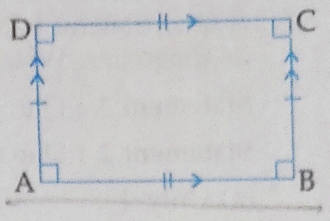
$\Rightarrow \quad x = 360° - 290°$

$\Rightarrow \quad x = 70°$

The fourth angle is 70°.

TYPES OF QUADRILATERALS

1. **Rectangle** : A quadrilateral in which opposite sides are equal and parallel and each angle is 90° is called a **rectangle.**



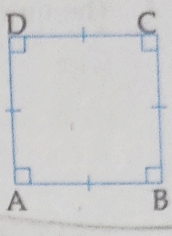
In the given figure ABCD is a rectangle in which

$AB = DC$; $AB \parallel DC$

$AD = BC$; $AD \parallel BC$

$\angle A = \angle B = \angle C = \angle D = 90°$

2. **Square** : A rectangle in which all four sides are equal is called a **square.**

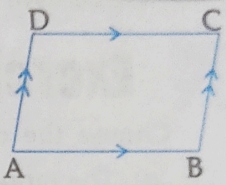


In the given figure, ABCD is square in which

$AB = BC = CD = DA$

$\angle A = \angle B = \angle C = \angle D = 90°$

3. **Parallelogram** : A quadrilateral in which both pairs of opposite sides are parallel is called a **parallelogram.**



In the given figure, ABCD is a parallelogram in which

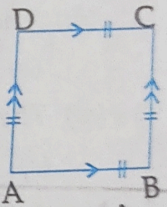
$AB = DC$, $AB \parallel DC$

and $AD = BC$, $AD \parallel BC$

Think and Answer

If you connect the pairs of diagonally opposite holes on a carrom board, what will be the measure of the four angles formed at the intersection?

4. **Rhombus** : A parallelogram in which all sides are equal is called a **rhombus.**

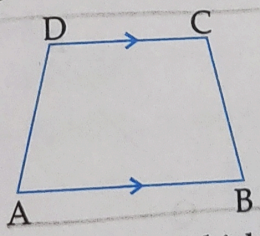


In the given figure, ABCD is a rhombus in which

$AB \parallel DC$; $AD \parallel BC$

and $AB = BC = CD = DA$

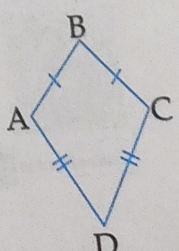
5. **Trapezium** : A quadrilateral having exactly one pair of parallel sides is called a **trapezium.**



In the given figure, ABCD is a trapezium in which $AB \parallel DC$.

A trapezium is said to be an **isosceles trapezium** if its non-parallel sides are equal *i.e.*, $AD = BC$.

6. **Kite** : A quadrilateral is a **kite**, if it has two pairs of equal adjacent sides. Its opposite sides are unequal.



In the given figure, ABCD is a kite.

Where, $AB = BC$, $AD = CD$

but $AD \neq BC$ and $AB \neq DC$.